Concurrent Engineering of Tolerance Synthesis and Process Selection for Products with Multiple Quality Characteristcs Considering Process Capability

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CONCURRENT ENGINEERING OF TOLERANCE SYNTHESIS AND PROCESS SELECTION FOR PRODUCTS WITH MULTIPLE QUALITY CHARACTERISTCS CONSIDERING PROCESS CAPABILITY

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Abstract

The existences of variances that are very difficult to be removed from manufacturing processes provide significance of tolerance to the product quality characteristics target of customer functional requirement. Furthermore, quality loss incurred due to deviation of quality characteristics of the target with a specified tolerance. This article discusses the development of concurrent engineering optimization model of tolerance design and manufacturing process selection on product with multiple quality characteristics by minimizing total costs in the system, namely total manufacturing cost and quality loss cost as functions of tolerance, also rework and scrap costs. The considered multiple quality characteristics have interrelated tolerance chain. The formulation of proposed model is using mixed integer non linear programming as the method of solution finding. In order to validate of the model, this study presents a numerical example. It was found that optimal solution are achieved from proposed model in the numerical example.

Abstrak

Rekayasa Simultan Sintesis Toleransi dan Pemilihan Proses untuk Produk dengan Multi-karakteristik Kualitas yang Mempertimbangkan Kapabilitas Proses. Keberadaan variansi yang sangat sulit untuk dihilangkan dalam proses manufaktur memberikan peran penting adanya toleransi terhadap target karakteristik kualitas produk yang menjadi kebutuhan fungsional bagi konsumen. Selanjutnya timbul kerugian kualitas yang disebabkan oleh penyimpangan karakteristik kualitas dari target dengan toleransi yang ditetapkan. Makalah ini membahas pengembangan model optimisasi rekayasa simultan desain toleransi dan pemilihan proses manufaktur pada produk dengan multi karakteristik kualitas untuk meminimasi total ongkos dalam sistem, yaitu total ongkos manufaktur dan ongkos kerugian kualitas yang merupakan fungsi dari toleransi serta ongkos *rework* dan ongkos *scrap*. Karakteristik kualitas produk yang diperhatikan dalam penelitian ini memiliki rantai toleransi yang saling berkaitan (*interrelated chain*). Formulasi model yang dikembangkan menggunakan *mixed integer non linear programming* sebagai metode pencarian solusi.

Keywords: optimization model, quality loss, rework, scrap, tolerance

1. Introduction

Tolerance synthesis is a critical issue in design and manufacturing stages, which affects both of product and process design because the tolerance is the bridge between product requirements and manufacturing cost [1]. Tolerance synthesis, which is also called tolerance allocation, is carried out in a direction opposite to tolerance analysis; from the tolerance of the function of interest to the individual tolerances [2]. According to Gryna *et al*. [3] tolerance is set by the engineering design function to define minimum and maximum values allowable for the product to work properly. In quality engineering, tolerance allocation reflects a trade-off between customer requirements and the ability of producers to satisfy them. Thus, the loss of both customers and producers can be balanced.

Tolerance synthesis in the product design phase focuses on efforts to meet products functional requirements with the tolerance values as tight as possible. Due to tight tolerance desired in the design phase, it is usually less

considering the process capability of production. While tolerance synthesis at the process design/planning phase is more focused on the ease in performing the production process. Thus, in this stage the desired tolerances allocation is set as loose as possible. Meanwhile, during a production process, variance leads to imperfections of the product. Therefore, tolerance synthesis must be considered in both product and process design.

The aim of tolerance synthesis is to determine the maximum and minimum deviation limits of the product quality characteristics due to imperfections of the production process. Costs consideration and the capability of production process to tolerance allocation have made the problem become more complex. The tolerance values will affect the ability of components to be assembled into a final product (assembly), process selection, tooling, set-up costs, operator skills, inspection and measurement, scrap and rework. Loose tolerance facilitates implementation of the production process resulting in lower production costs. Consequently, if tolerance is too loose it will reduces the quality performance of the product. On the other hand, strict tolerance will improve product performance, but scrap and rework costs are higher. A lower process capability causes more deviation of quality characteristics from maximum and minimum limits. Thus, allocation of optimal tolerance values should take into account to quality loss and manufacturing cost by considering design and tolerance constraints and also other constraints [4]. Quality loss function has been introduced by Taguchi [5] in order to make it easy for designer to make trade-off between manufacturing and quality loss cost. Moreover, process (machine) selection problem in the planning process can be done by minimizing total cost of manufacturing and quality loss cost.

A major focus in tolerance design model was for a single quality characteristic by incorporating quality loss [4,6-9]. Recently, Mustajib and Irianto [10] propposed an integrated model for tolerance allocation and selection process in multi-stage processes. Meanwhile, the importance of determining tolerances for multiple quality characteristics products has been demonstrated by Lee and Tang [11], Chou and Chen [12], Huang *et al.* [13], and Peng *et al.* [14]. Furthermore, many researches [15-17] extended this work on assembly products with multiple quality characteristics based on statistical approach.

Statistical approach estimates the accumulation of tolerance (stack-up condition) on the assembled product, which is based on the fact that the probability of the component is at an extreme lapse very low tolerance [18]. The impact of this is not only on tighter tolerance assembled product with looser component

tolerances but also lower manufacturing cost. On the other hand, when the precision limit of the process and tolerance limits of final product are based on worst-case criteria it tends to require tighter component tolerances with relatively expensive manufacturing cost, compared with statistical approaches (root sum square criteria).

Although statistical approaches improved performance, many researchers have not addressed the problem of process selection by considering process capability. Process capability indices, including Cp, Cpk, and Cpm, have been proposed in the manufacturing industry to provide numerical measures on whether a process is capable of reproducing items meeting the manufacturing quality requirement preset in the factory [19]. In addition, tolerance allocation could minimize rework and scrap costs, but most of the researches in tolerance synthesis did not consider the scrap and rework rates of the process resulted from allocated tolerance.

Based on those shortcomings of existing model of tolerance synthesis of multiple quality characteristics in previous studies and the need for reducing manufacturing cost, we propose the development of concurrent engineering optimization model of tolerance synthesis and process selection by considering process capability and non conformance rate; scrap and rework rates. The overall objective of the model is minimizing total cost of the system, i.e. manufacturing and quality costs. The development was carried out by enhancing on the model of previous study [8,9,11-14] with respect to process capability and statistical approaches, also taking into account the quality loss cost is a function of tolerance as well as rework and scrap cost.

2. Methods

Taguchi in Taguchi *et al.* [5] defined quality of a product as the minimum product loss imparted by the society from the time product shipped. Loss due to the characteristics of the product quality cannot meet customer needs and satisfaction. Taguchi argues that there is a loss (in the form of cost and quality) when the quality characteristic deviating from the target product, although the quality characteristic deviating from the target is still within the specifications or the specified tolerances. The losses arise because of the waste, loss of opportunity (opportunity cost) and cost when the product fails to meet the target value specified by quality characteristics. From the producers' view, these losses can be quantified in the form of costs incurred by producers of quality; the cost of rework and scrap, cost of inspection. From customers's view, these losses are form of disatisfaction of the product variability received. Based on the variability of the product quality, it can be quantified in monetary terms. Moreover, the quantification of the quality loss cost can be done by using Taguchi quadratic loss function approach, Eq. 1:

$$L(y) = r(y - m)^2 \qquad (1)$$

with $r = \dfrac{C}{t^2}$ (2)

In Eq.1 r is a constant that converts the characteristics become the characteristics of engineering costs which is the loss coefficient of the final product quality. This costant is estimated based on the cost of rework (C) where is required when quality characteristics of the final product y deviating from the target m, but still within an acceptable tolerance limits customers (t). Further determination of the optimal value of tolerance should pay attention to aspects of the quality loss cost $L(t)$ and the manufacturing cost $P(t)$ with respect to design constraints and tolerance of other constraints that are relevant. In Fig. 1, $L(t)$ states the cost of quality loss as a function of tolerance, whereas $P(t)$ states the cost of manufacturing as a function of tolerance. Furthermore, $P(t)$ also states the overall cost of manufacturing on the quality characteristics with tolerance t.

Quality Characteristics. The problem, however, in Eq. 1 above is applied only for products with single quality characteristic, while for products with multiple quality characteristics the value of quality loss can be expanded [9]:

$$L(Y) = (Y - M)^T R(Y - M) \qquad (3)$$

Eq. 2 is general; it means that each of quality characteristics are not considered whether it is correlated or not. In Eq. 3 above, $Y=[Y_1,...Y_q,...Y_Q]^T$ states the vector for the qth quality characteristic and the vector of the qth quality characteristic target is denoted by $M=[M_1,...M_q,...M_Q]^T$, and R is $r \times r$ matrix for constant of the qth quality characteristic.

The value of the qth quality characteristic (Y_q) can be estimated from the dimension of ith component. This relationship can be stated in Eq. [4]:
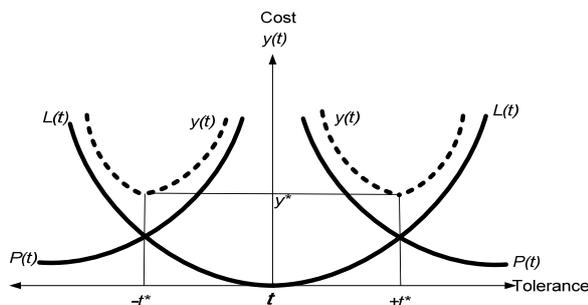
$$Y_q = f_q(K_1,...K_i...K_I). \qquad (4)$$

Figure 1. The Solution Space for an Unconstrained Space [4]

The mean of nominal dimension of the ith component (K_i) is $\mu_1,...\mu_i,...\mu_I$. Expanding of right-hand side Eq. 4 in Taylor series around $\mu_1,...\mu_i,...\mu_I$, by ignoring the higher order we obtained [21]:

$$Y_q = M_q + \sum_{i=1}^{I}\left(\left.\frac{\partial Y_q}{\partial K_i}\right|_{\mu_i}\Delta K_i\right) \qquad (5)$$

This study consider that each of components of the ith component (K_i) can be produced by alternative processes $1,...j,...J$, so that quality characteristic $Y_q=f_q(K_{11},...K_{ij},...K_{IJ})$ in Eq. 5 become:

$$Y_q = M_q + \sum_{i=1}^{I}\sum_{j=1}^{J}\left(\left.\frac{\partial Y_q}{\partial K_{ij}}\right|_{\mu_i}\Delta K_{ij}\right) \qquad (6)$$

Furthermore, deviation of of the qth quality characteristic (Dq) can be estimated based on the difference between the qth quality characteristic (Y_q) with the qth quality characteristic target (Mq):

$$D_q = Y_q - M_q = \sum_{i=1}^{I}\left(\left.\frac{\partial Y_q}{\partial K_i}\right|_{\mu_i}\Delta K_i\right) \qquad (7)$$

Thus, the vector of multiple quality characteristics deviation is $D=[D_1,...D_q,...D_Q]^T$. Based on the vector of multiple quality characteristics deviation, Eq. 3 can be written with Eq. 8 to evaluate the multivariate quality loss function with:

$$L(D) = D^T RD \qquad (8)$$

or

$$C = \underline{D}^T R\underline{D} \qquad (9)$$

Elements of R matrix can be calculated based on a basic correlation in Eq. 2 that are extended to be loss of multiple quality characteristics. This expansion is formulated with [9]:

$$\sum_{q=1}^{p}\sum_{i=1}^{p} R_{qi} D_q^{(r)} D_i^{(r)} = C_r , \qquad (10)$$

with $r = 1, 2, \ldots, p(p+1)/2$

R_{qi} represents the qth row and the ith column in matrix elements of R. When both set of product quality deviation of D (denoted by $D_q^{(r)}$) and vector of $D_i^{(r)}$ and quality loss cost (Cr) are known, then the matrix elements of R (denoted by R_{qi}) can be finded. In case $p=2$, it is required three data sets for matrix elements.

Let we consider again that Eq. 7 can be expressed as the variance of the qth quality characteristic (σ_q^2) with:

$$\sigma_q^2 = \sum_{i=1}^{I}\sum_{j=1}^{J}\left(\left.\frac{\partial Y_q}{\partial K_{ij}}\right|_{\mu_{ij}}\right)^2 \Delta K_{ij}^2 \qquad (11)$$

ΔK_i^2 in Eq. 11 is nothing but the variance of the *i*th component dimension that produced by the *j*th process (σ_{ij}^2) which is statistically can be written with:

$$\sigma_q^2 = \sum_{i=1}^{I} \sum_{j=1}^{J} \left(\left. \frac{\partial Y_q}{\partial K_{ij}} \right|_{\mu_{ij}} \right)^2 \sigma_{ij}^2 \qquad (12)$$

Eq. 12 is in accordance with the statistical criteria approach (root sum square criteria) which states that the dimension of assembly product consists of variance dimensional of its components. Furthermore, if we consider that the *q*th quality characteristic derived from one or more components of interrelated dimensions chain, then the covariance of the *r*th and *r*th quality characteristic can be finded by Eq. 13:

$$Cov\left(\sigma_r^2, \sigma_s^2\right) = \sum_{i=1}^{I} \sum_{j=1}^{J} \left(\left. \frac{\partial Y_q}{\partial K_{ij}} \right|_{\mu_{ij}} \left. \frac{\partial Y_s}{\partial K_{ij}} \right|_{\mu_{ij}} \right)^2 \sigma_{ij}^2 \quad (13)$$

Variance for the *i*th component produced by the *j*th process (σ_{ij}^2) in Eq. 12 and 13 can be estimated by considering process capability index (*Cp*):

$$Cp_i = \frac{USL_i - LSL_i}{6\sigma_i} \qquad (14)$$

Where are *USLi* and *LSLi* state upper and lower specification limits of the *i*th component. By considering the tolerance of one side only, Eq. 14 can be simplified to obtain variances and relationships to the *j*th component tolerance (t_j) in Eq 15:

$$\sigma_{ij}^2(t_{ij}) = \left(\frac{t_{ij}}{3Cp_{ij}} \right)^2 \qquad (15)$$

This research consider that on average process have a mean shift of 1.5σ. It is consistent with Motorola's original Six Sigma program which stipulate that a process was said to be six sigma quality level when *Cp*=2 and *Cpk*=1.5. By assuming 1.5σ process shift, Taguchi process capability index can be obtained [20]:

$$Cpm = \frac{Cp}{\sqrt{1 - \left(\frac{1.5\sigma}{\sigma} \right)^2}} = 0.5447Cp \qquad (16)$$

Moreover, the variance of the *q*th quality characteristic in the Eq. 12 and 13 can be rewritten into Eq. 17 and 18 by substituting Eq. 15 into Eq. 12 and 13 as follows.

$$\sigma_q^2(t_{ij}) = \sum_{i=1}^{I} \sum_{i=1}^{I} \left(\left. \frac{\partial Y_q}{\partial K_{ij}} \right|_{\tau_{ij}} \right)^2 \left(\frac{t_{ij}}{5.51Cpm_{ij}} \right)^2, \forall q \in Q \quad (17)$$

$$Cov\left(\sigma_r^2, \sigma_s^2\right) = \sum_{i=1}^{I} \sum_{j=1}^{J} \left(\left. \frac{\partial Y_q}{\partial K_{ij}} \right|_{\mu_i} \left. \frac{\partial Y_s}{\partial K_{ij}} \right|_{\mu_{ij}} \right)^2 \left(\frac{t_{ij}}{5.51Cpm_{ij}} \right)^2 \quad (18)$$

Meanwhile, the variance covariance matrix can be expressed by σ_q^2 vector of parameter [11]:

$$V\sigma_q^2(t) = \begin{bmatrix} \sigma_1^2 & Cov\left(\sigma_1^2, \sigma_2^2\right) & \cdots & Cov\left(\sigma_1^2, \sigma_p^2\right) \\ Cov\left(\sigma_1^2, \sigma_2^2\right) & \sigma_2^2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ Cov\left(\sigma_1^2, \sigma_p^2\right) & \cdots & \cdots & \sigma_p^2 \end{bmatrix} \quad (19)$$

Quality loss cost. Expected cost of quality loss due to deviation of quality characteristic from its target can be estimated by tracking the variance covariance matrix [11]. Thus, the quality loss expectations in Eq. 3 can be calculated by tracing the variance covariance matrix based on Eq. 17 which can be formulated in Eq. 20.

$$E\left[L(t_{ij})\right] = Trace\left[RV\sigma_q^2(t_{ij})\right] \qquad (20)$$

Cost of non conformance. The process alternatives and tolerance allocation have possibility to produce non fonformance if the process variances are out from minimum and maximum values allowed. Costs of non conformance includes rework and scrap cost. By assuming that all variance for the *i*th component produced by the *j*th process are normally distributed, then the probability of non concurrence ($\bar{g}_{ij}$) is determined by probability meets tolerance limits (g_{ij}):
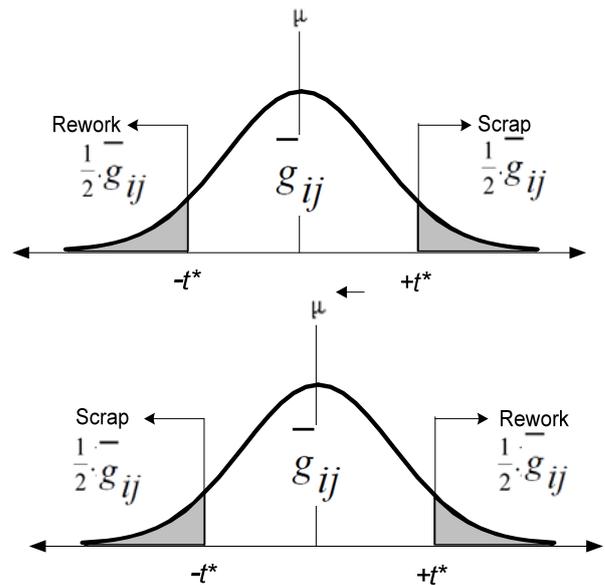
$$\bar{g}_{ij} = 1 - g_{ij} \qquad (21)$$

Figure 2. Standarized Process Tolerances

If the product is not to meets tolerance ($\bar{g}_{ij}$) is classified into two types, namely: (1) $\bar{g}_{1ij}$, for probability where tolerance is not met owing to undersize, and (2) $\bar{g}_{2ij}$, for probability where tolerance is not met due to oversize.

Thus, for symmetric bilateral tolerance (see Fig 2), both of rework rate and scrap rate can be obtained by following equation:

$$\bar{g}_{1ij} = \bar{g}_{2ij} = \frac{1}{2} \cdot \bar{g}_{ij} \qquad (22)$$

Model formulation. The objective function the model is minimizing total costs (*TC*), which is the sum of manufacturing cost and quality costs. Quality costs includes quality loss cost and failure costs. The cost of failure arises when the dimensional tolerance cannot be met and results in component dimensions are undersized or oversized. In mechanical product, if the dimension is undersize at the features likes a hole, a step, a groove, and a slot, it is need to be reworked with rework cost, c^r_{ij}. On the other hand, if the dimension is oversize, it is need to be scrapped with certain of scrap cost, c^s_{ij}. If the dimensional tolerance is bilateral, then probability process cannot meets the dimensional tolerance ($\bar{g}_{ij}$) due to oversized is equal to probability cannot meets the dimensional tolerance due to undersized component [22]. Therefore, complete model of the objective function can be expressed by Eq. 23. Meanwhile, all of the model constraints are formulated by Eq. 24 to 27 respectively.

$$Min\ TC = \begin{bmatrix} \sum_{i=1}^{I}\sum_{j=1}^{J}\left(A_{ij}e^{-B_{ij}t_{ij}} + C_{ij}\right)X_{ij} + \\ Trace\left[RV\sigma_q^2\left(t_{ij}\right)\right]X_{ij} + \\ \sum_{i=1}^{I}\sum_{j=1}^{J}C_{ij}^r\bar{g}_{ij}^r X_{ij} + \sum_{i=1}^{I}\sum_{j=1}^{J}C_{ij}^s\bar{g}_{ij}^s X_{ij} \end{bmatrix} \qquad (23)$$

subject to:
1) Quality spesification of design

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\left(\frac{\partial Y_q}{\partial K_{ij}}\right)^2 \frac{t_{ij}^2}{Cpm_{ij}} x_{ij} \le \frac{t_{Yq}^2}{Cpm_{Yq}},\ \forall_q \in Q \qquad (24)$$

2) Process precision limits

$$t_{ij}^{min} \le t_{ij} \le t_{ij}^{max},\ \forall_i \in I\ \forall_j \in J \qquad (25)$$

3) Alternative process

$$\sum_{J=1}^{J} X_{ij} = 1, \forall_i \in I \qquad (26)$$

4) Binary decision variables

$$X_{ij} \in [0,1], \forall_i \in I, \forall_j \in J \qquad (27)$$

Notations

i. Indexs:

q = *q*th quality characteristic
i = *i*th component
j = *j*th process alternative
Q = set of quality characteristics
I = set of components
J = set of process alternatives

ii. Decision variables:

X_{ij} = 1, if the *i*th component is processed by using *j*th process alternative. And 0, if not.

t_{ij} = tolerance of the *i*th component which is produced by using *j*th process alternative

iii. Performance:

TC = total costs

iv. Parameters:

A_{ij}, B_{ij}, C_{ij} = coefficients of cost-process tolerance function for the *i*th component by using *j*th process alternative to generate tolerance t_{ij}

R = *p x p* positive definte matrix for quality loss constant the *q*th quality characteristic

$V\sigma_q^2$ = vector parameter σ_q^2 of the variance covariance matrix

σ_{ij}^2 = variance for the *i*th component produced by the *j*th process

Y_q = *q*th quality characteristic

$\dfrac{\partial Y_q}{\partial K_{ij}}$ = partial derivative of *q*th quality characteristic with respect to *i*th component produced by the *j*th process

Cpm_{ij} = Taguchi process capability index for the *i*th component produced by the *j*th process

Cpm_{Yq} = Taguchi process capability index for the *q*th quality characteristic

c_{ij}^r = rework cost for the *i*th component produced by the *j*th process

c_{ij}^s = scrap cost for for the *i*th component produced by the *j*th process

t_{Yq} = tolerance design of *q*th quality characteristic

t_{ij}^{max} = upper tolerance limit for the *i*th component produced by the *j*th process

t_{ij}^{min} = lower tolerance limit for the *i*th component produced by the *j*th process

$\bar{g}_{ij}^s$ = scrap rate of non conforming to specification for the *i*th component produced by the *j*th process.

$\bar{g}_{ij}^r$ = rework rate of non conforming to specification for the *i*th component produced by the *j*th process.

3. Results and Discussion

This section presents numerical example which is related to quality characteristic of one mechanical product, helical spring. This product is a type of compression spring made of round wire (diameter, d_w), wrapped in a straight line (length, L), cylindrical form with outer diameter D_o and the number of coil springs is n with constant distance between one coil with the others coils (Fig. 3). The use of helical spring is often found in a variety of equipments. In motor vehicles, the springs are usually used in suspension systems, engine valve springs and the clutch plate. Meanwhile, in the manufacturing process it is often used for the striper and the control valve of hydraulic and pneumatic systems. The springs are also widely used in small appliances such as on electrical switches, pen ballpoint, etc.

The main performance of a helical spring is an aspect that must be met within the design specifications. The purpose of helical spring design is to determine the dimensions of spring that can be operated at the limit load (F) and certain axial deflection (ΔL). Allowable stress depends on the material and dimensions of helical spring. Thus, the purpose of a helical spring design is quality specifications that are expected by the designers as customer needs.

Compressive force imposed axially on the helical spring (F) will cause axial deflection (ΔL). The relationship between the resulted force and deflection
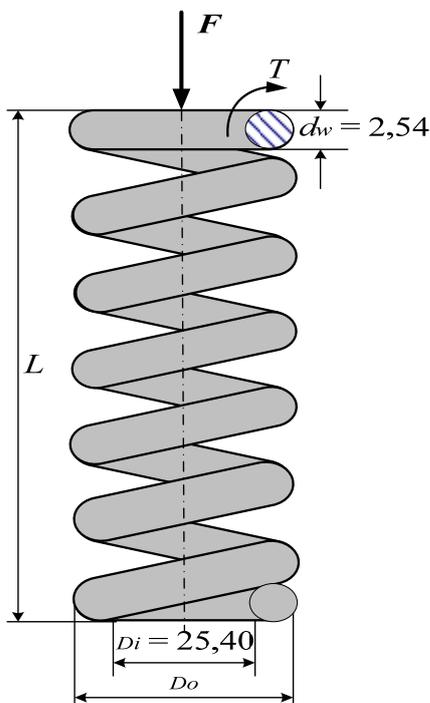

Figure 3. Helical Spring

is called spring constant (k) or spring stiffness. Spring constant can be calculated by dividing the changing force and deflection.

$$k = \frac{F}{\Delta L} \tag{28}$$

Because the loading is transmitted through wire spring, it will cause torsion. Therefore, the stress arising in the wire is the torsion shear stress (τ) which can be derived from the classical equation [23]:

$$\tau = \frac{Tc}{J_p} \tag{29}$$

Meanwhile, the angular deflection (θ) on the wire can be calculated by using Eq. 30

$$\theta = \frac{TL}{GJ_p} = \frac{\left(F\dfrac{D_o}{2}\right)\pi D_o n}{\dfrac{\pi}{32}d_w^4 G} = \frac{16FD_o^2 n}{d_w^4 G} \tag{30}$$

Where T is the applied torque, G is the modulus of elasticity of spring material, and Jp states moment of polar inertia of wire material. Furthermore, axial deflection (ΔL) can be obtained by Eq. 31.

$$\Delta L = \theta \frac{D_o}{2} \tag{31}$$

Note that the precision of spring outer diameter (D_o) is influenced by the precision of spring internal diameter (D_i) and the spring wire diameter (d_w). We consider that the quality characteristic of outer diameter helical spring (D_o) can be formulated by Eq. 32.

$$D_o = D_i + 2d_w \tag{32}$$

By looking back to Eq. 28 as the basic equation represents spring performance, then the next quality characteristic helical spring which states the spring constant (k) or the spring stiffness can be formulated by Eq. 33 through substitution the value of θ in Eq. 28 and 32 into Eq. 31

$$k = \frac{Gd_w^4}{8(D_i + d_w)^3 n} \tag{33}$$

Thus, both Eq. 32 and 33 above are the formulations for two quality characteristics of the helical spring. Furthermore, D_0 is called the quality characteristic of Y_1 and k is called the quality characteristic of Y_2.

Partial derivative quality characteristic of Y_1 and Y_2 to the component dimensions D_i, d_w, and n which is resulted by process alternative (see Table 1) can be calculated as follows.

$$\frac{\partial Y_1}{\partial K_1} = \frac{\partial D_0}{\partial D_i} = 1 \tag{34}$$

Table 1. Cost Parameters, Tolerance Limits, and Non Conformance Rates

Dimension	Process Alternative	Cost Parameter			Lower Tolerance	Upper Tolerance	Scrap Rate	Rework Rate
		A	B	C				
d^w	1	1.581	78.735	1.44	0.018	0.8	0.003	0.001
	2	14.132	7.262	1.44	0.02	0.82	0.002	0.002
D^i	1	17.364	39.333	0.5	0.218	1.2	0.007	0.003
	2	78.735	3.124	0.55	0.23	1.26	0.006	0.002
n	1	65.634	214.097	1.5	0.22	0.26	0.002	0.004
	2	61.138	20.682	1.5	0.22	0.25	0.004	0.003

$$\frac{\partial Y_1}{\partial K_2} = \frac{\partial D_0}{\partial d_w} = 2 \tag{35}$$

In similar manner, the partial derivatives for Y_2 are as follows:

$$\frac{\partial Y_2}{\partial K_1} = \frac{\partial k}{\partial D_i} = -\frac{37500 d_w^4}{n(D_i + d_w)^4} = -0.256 \tag{36}$$

$$\frac{\partial Y_2}{\partial K_2} = \frac{\partial k}{\partial d_w} = \frac{50000 d_w^3}{n(D_i + d_w)^3} - \frac{37500 d_w^4}{n(D_i + d_w)^4} = 3.5 \tag{37}$$

$$\frac{\partial Y_2}{\partial K_3} = \frac{\partial k}{\partial n} = -\frac{12500 d_w^4}{n^4(D_i + d_w)^3} = -0.238 \tag{38}$$

The data obtained from previous study [12], for example, are the number of coils ($n \approx 10$), modulus of elasticity of helical spring material, $G = 100.000$ kg/mm, and quality tolerance of product design (Table 1). Suppose that the values of product design tolerances are D_0=1.107 and k=1.106. Furthermore, the quality characteristic of Y_1 and Y_2 deviate from their target vector by values:

$$\underline{D}^{(1)} = \left[\underline{D}_1^{(1)}, \quad 0\right]^T = [0.88, \quad 0]^T \tag{39}$$

$$\underline{D}^{(2)} = \left[0, \quad \underline{D}_2^{(2)}\right]^T = [0, \quad 0.77]^T \tag{40}$$

$$\underline{D}^{(3)} = \left[\underline{D}_1^{(3)}, \quad \underline{D}_2^{(3)}\right]^T = [0.58, \quad 0.47]^T \tag{41}$$

will result in product failure and cause a loss of 60\$.

Furthermore, elements of the quality loss constant matrix can be obtained through Eq. 10.

$$R_{11} = \frac{C_1}{\left(\underline{D}_1^{(1)}\right)^2} = \frac{60}{0.88^2} = 77.48 \tag{42}$$

$$R_{22} = \frac{C_2}{\left(\underline{D}_2^{(2)}\right)^2} = \frac{60}{0.77^2} = 101.20 \tag{43}$$

Table 2. The Optimal Solutions of Three Process Capability Scenarios

	Cpm		
	0.888	1	1.109
t_1	0.031	0.032	0.032
t_2	0.218	0.23	0.218
t_3	0.22	0.22	0.22
X_1	1	2	1
X_2	1	2	1
X_3	1	1	1
TC	19.40203	18.4911	16.38516

$$R_{12} = R_{21}$$
$$= \frac{\left[C_3 - C_1\left(\underline{D}_1^3\right)^2 \Big/ \left(\underline{D}_1^1\right)^2 - C_1\left(\underline{D}_2^3\right)^2 \Big/ \left(\underline{D}_2^2\right)^2\right]}{\left(2\underline{D}_1^3 \underline{D}_2^{(3)}\right)} \tag{44}$$
$$= \frac{\left[60 - 60(0.58)^2 \big/ (0.88)^2 - 60(0.47)^2 \big/ 0.77^2\right]}{(2 \times 0.58 \times 0.47)} = 21.24$$

Meanwhile, elements of the variance covariance matrix can be obtained through Eq. 17 and 18. In case *Cpm*=1, the expected of quality lost cost can be generated by using of MathCAD 14.0 software as follows:

$$E\left[L\left(t_{ij}\right)\right] = 71.36 t_{11}^2 + 73.93 t_{12}^2 + 144.66 t_{21}^2 + \\ 147.23 t_{22}^2 + 123.72 t_{31}^2 + 123.72 t_{32}^2 \tag{45}$$

Finally, decision variables of the optimization model of tolerance design and manufacturing process selection on helical spring can be obtained by solved in the proposed model by using the LINGO software package. The optimal tolerances and process with another two process capability scenarios are given in Table 2.

The optimization result data in Table 2 indicates a more reasonable relationship between process capability and total costs in the system. As can be seen, the impact of process capability increases has significantly reduced total costs. On the other hand, the changes of process capability have changed tolerances allocation and selected processes.

4. Conclusion

This study proposed the development of concurrent engineering optimization model of tolerance synthesis and process selection by considering process capability and costs of non conformance. The objective of the model is minimizing total cost of the system, i.e. manufacturing and quality costs. Formulation of the model developed using mixed integer non linear programming as the method of solution finding. In order to validate of the developed model, this study presents a numerical example. It was found that optimal solution are resulted from proposed model in the numerical example. The optimization results data in indicate that there were relationship between process capability and total costs in the system. Meanwhile, the impact of process capability increasing have significantly reduced total cost. Moreover, the changes of process capability have changed tolerance allocation and selected process. In summarize, this finding is promising and should be explored with other methods for finding better optimal solutions. It is possible to use computational intelligence to enhance the method of solution finding for proposed model.

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